An Empirical Study on Optimal Reinsurance about crop insurance in China
—–Based on data from Inner Mongolia, Jilin and Liaoning

Yunbo Wang
AnHua Agricultural Insurance Company, Beijing, China.
Outline

• Motivation
• Model and data
• Results
• Conclusion
Motivation

• Agricultural Insurance
  – Adverse weather, flood, draught, hail etc.
  – Diseases of livestock.
  – Resembles catastrophe insurance.
  – Reinsurance is a major risk transfer instrument.

• Agricultural Insurance in China
  – The world's second-largest agricultural insurance market.
  – Crop insurance accounts for over 90%.
Motivation

• Reinsurance of crop insurance during 2008-2012
  – Cumulated original premiums: 62.68 billion RMB
  – Cumulated ceded premiums: 9.50 billion RMB.
  – Cumulated losses recovered from reinsurers: 4.57 billion RMB.
  – Cumulated expenses recovered from reinsurers: 2.33 billion RMB.
  – Cumulated net ceded profit: 2.13 billion RMB.
  – Ceded profit accounts for 3.4% of original premiums.
  – Ceded profit accounts for 22.42% of ceded premiums.
Motivation

• Considering the high cost of reinsurance for crop insurance:
  – How to determine the optimal reinsurance arrangement?
  – And what is the appropriate cost for it?
Model and data

Empirical model:

\[
\begin{align*}
\min_f & \quad \rho(x, f) \\
\text{s.t.} & \quad 0 \leq f_i \leq x_i \\
& \quad \pi(f) \leq \pi
\end{align*}
\]

- \( x = (x_1, x_2, \ldots, x_n) \), n-dimension samples, can be collected directly, or generated randomly.
- for loss \( x_i \), the insurer cedes \( f_i \) to a reinsurer.

- Empirical models can be transformed into Second Order Conic Programming problems (Weng, 2009)
- CVX MATLAB toolbox (Grant et al., 2013) can be used to solve this problem
Model and data

• Advantages of empirical model (Weng, 2009)
  – Simple, intuitive, practical.
  – It exploits directly the observed data.
  – Applies to a number of premium principles.

• Premium principles
  – Expectation principle: safety loading is unrelated to the variation of the risk:
    \[ \pi(f) = (1 + \beta)E[f(X)] \]
  – Standard deviation principle: safety loading is positively related to the variation of the risk:
    \[ \pi(f) = E[f(X)] + \beta \sqrt{Var[f(X)]} \]
Model and data

• Risk measures
  
  – $VaR_X(\alpha) = \inf \{x: \Pr(X > x) \leq \alpha\}$
  
  – $CTE_X(\alpha) = E[X|X > VaR_X(\alpha)]$

  – CTE is a better risk measure compared to VaR (Cai et al., 2008).

• In this study
  
  – To minimize the risk measure under the reinsurance premium budget constraint.
Model and data

- Other assumptions
  - Risk tolerance level $\alpha$: 2%, 5%, 10%, 20%.
  - Reinsurance premiums budget $\pi$: 3%, 3.75%, 4%, 5% of total premiums.
  - Safety loading coefficient $\beta$: 20%, 30%, 40%, 50%.
  - Samples size: 1000.
Model and data

• Loss ratio distribution (AHCRES)
  – Inner Mongolia: Gamma Distribution, \( a = 5.74987 \) and \( b = 0.105108 \).
  – Liaoning: Gamma Distribution, \( a = 4.1405 \) and \( b = 0.1796 \).
  – Jilin: Generalized Extreme Value Distribution, \( a = 0.105086 \), \( b = 0.173391 \) and \( c = 0.53431 \).

– Gamma Distribution: \( f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}} \)

– Generalized Extreme Value Distribution:
  \[
  f(x) = \left(\frac{1}{b}\right) \exp \left( - \left( 1 + a\frac{(x-c)}{b} \right)^{-\frac{1}{a}} \right) \left( 1 + a\frac{(x-c)}{b} \right)^{-1-\frac{1}{a}}
  \]
Results - Inner Mongolia

Fixed $\alpha$, varied $\pi$ and $\beta$ — scatter plots of $\{(x_i, f_i)\}$

- When $\pi$ is small, limited stop loss or its variants are optimal, with the increase of $\pi$, the optimal form may become stop loss without a limit.
- With the increase of $\beta$, the optimal form may become limited stop loss from stop loss.
- Small $\pi$ and large $\beta$ play similar roles in the process of optimal reinsurance design.
- The reality is, fairly large $\beta$ and small $\pi$ (strict reinsurance premiums budget).
Robustness test- Inner Mongolia

- For \( \{(x_i, f_i)\} \), fit \( f(x) = \min\{c(x - d)_+, m\} \), we replicate the random samples 500 times independently to obtain 500 independent estimates of \( \hat{c} \) and \( \hat{d} \) using \( \varepsilon = 0.001 \).

- For one simulation, if \( |f(x_i) - f_i| \leq \varepsilon \) for \( i = 1, 2, ..., 1000 \), one admission is obtained, admissibility=times of admission/500.

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## Results - Inner Mongolia

### Optimal reinsurance arrangement of Inner Mongolia under different constraints

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Conclusion

• When the primary insurer’s loss function, the reinsurance premium calculation principle, risk measure $\rho$ are given, $\alpha$, $\beta$, $\pi$ all affect the optimal reinsurance design.

• When strict constraint on reinsurance premiums budget $\pi$ are implemented (which is often the reality), Limited Stop Loss Reinsurance is optimal.

• Reinsurance premium calculation principle and safety loading coefficient of reinsurers play important roles in the optimal reinsurance decision-making process.
Thank you for your attention